



# **Mendelevian search for materials with optimal properties**

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And is based on the following papers:

- Z. Allahyari, A. R. Oganov. Nonempirical Definition of the Mendeleev Numbers: Organizing the Chemical Space. *J. Phys. Chem. C*, 124, 43, 23867–23878 (2020).
- Z. Allahyari, A. R. Oganov. Coevolutionary search for optimal materials in the space of all possible compounds. *NPJ Comput Mater*, 6, 55 (2020).
- Z. Allahyari, A. R. Oganov. Multi-objective Optimization as a Tool for Material Design. In: Andreoni W., Yip S. (eds) Handbook of Materials Modeling. Springer, Cham (2020).

## Outline

Search for materials with optimal target properties:

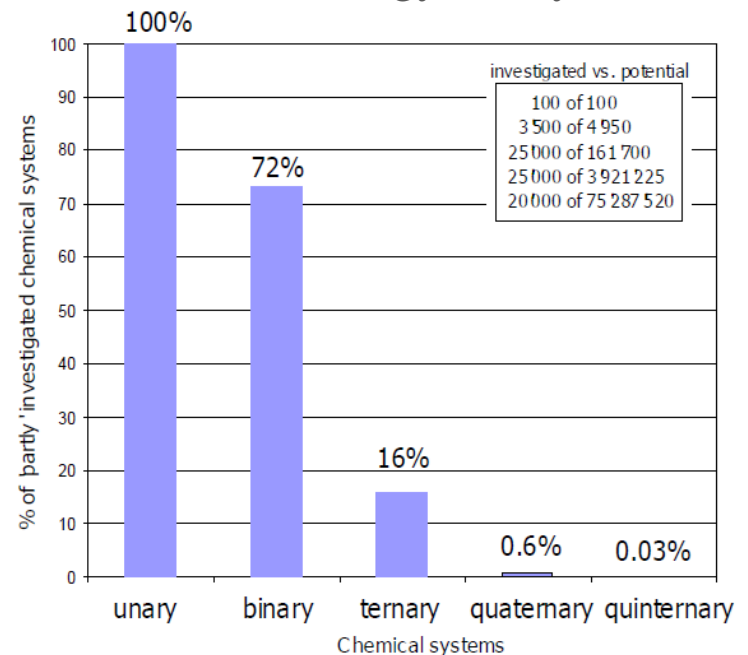
**Mendeleevian Serach-MendS** (algorithm, method)[1][2].

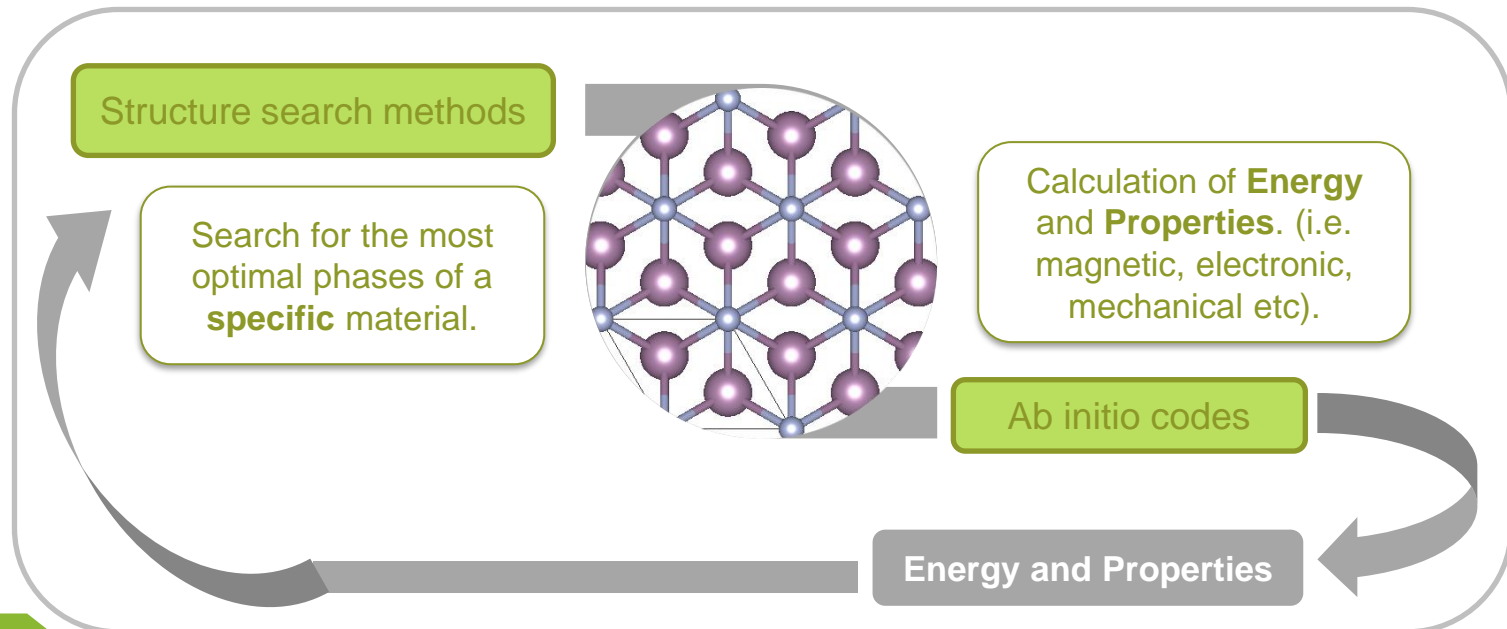
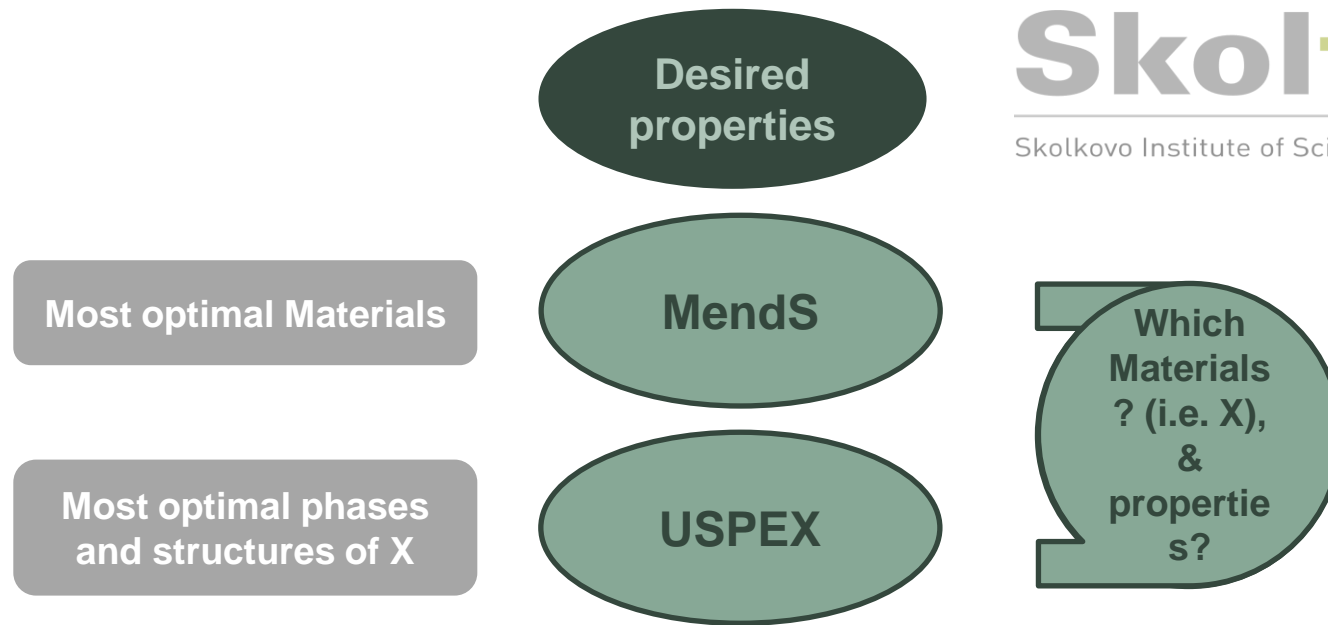
- Coevolutionary algorithm.
- A well-designed chemical space.
- Multi-objective Pareto technique.

**Results** on searching (using MendS) for hard/superhard and low energy binary materials under zero pressure in the entire chemical space.

[1] Patent No: WO2018009090A1

[2] *NPJ Comput. Mater.* **6**, 55 (2020).





# What is coevolution and what is the difference of evolutionary algorithm and coevolutionary algorithm?

EA- Giraffe( one species only)

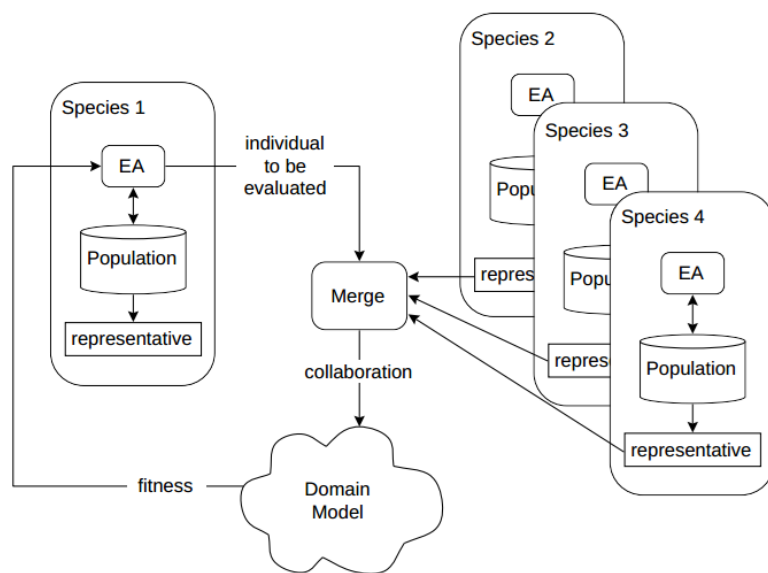
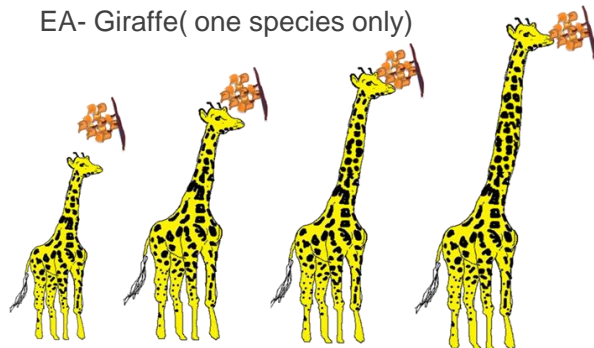
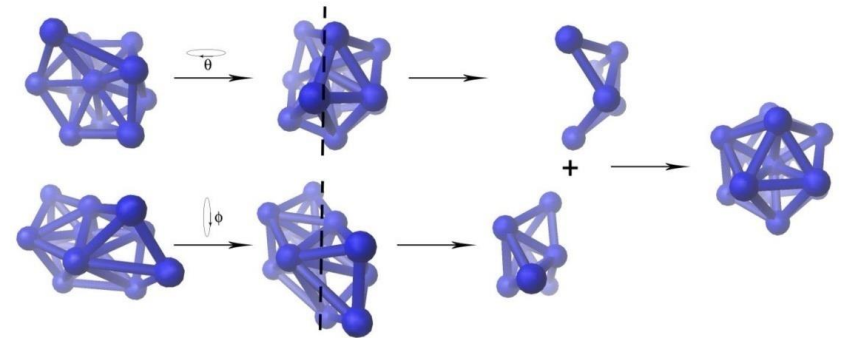
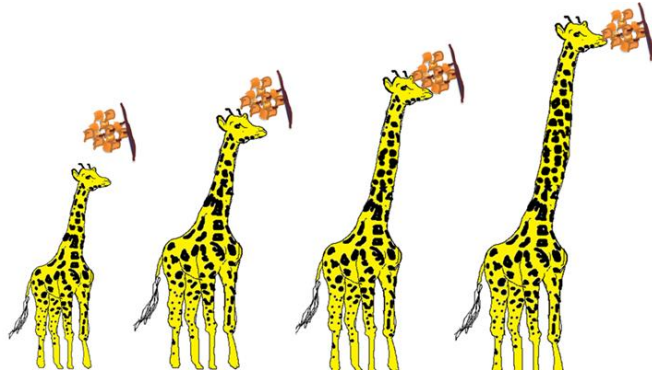


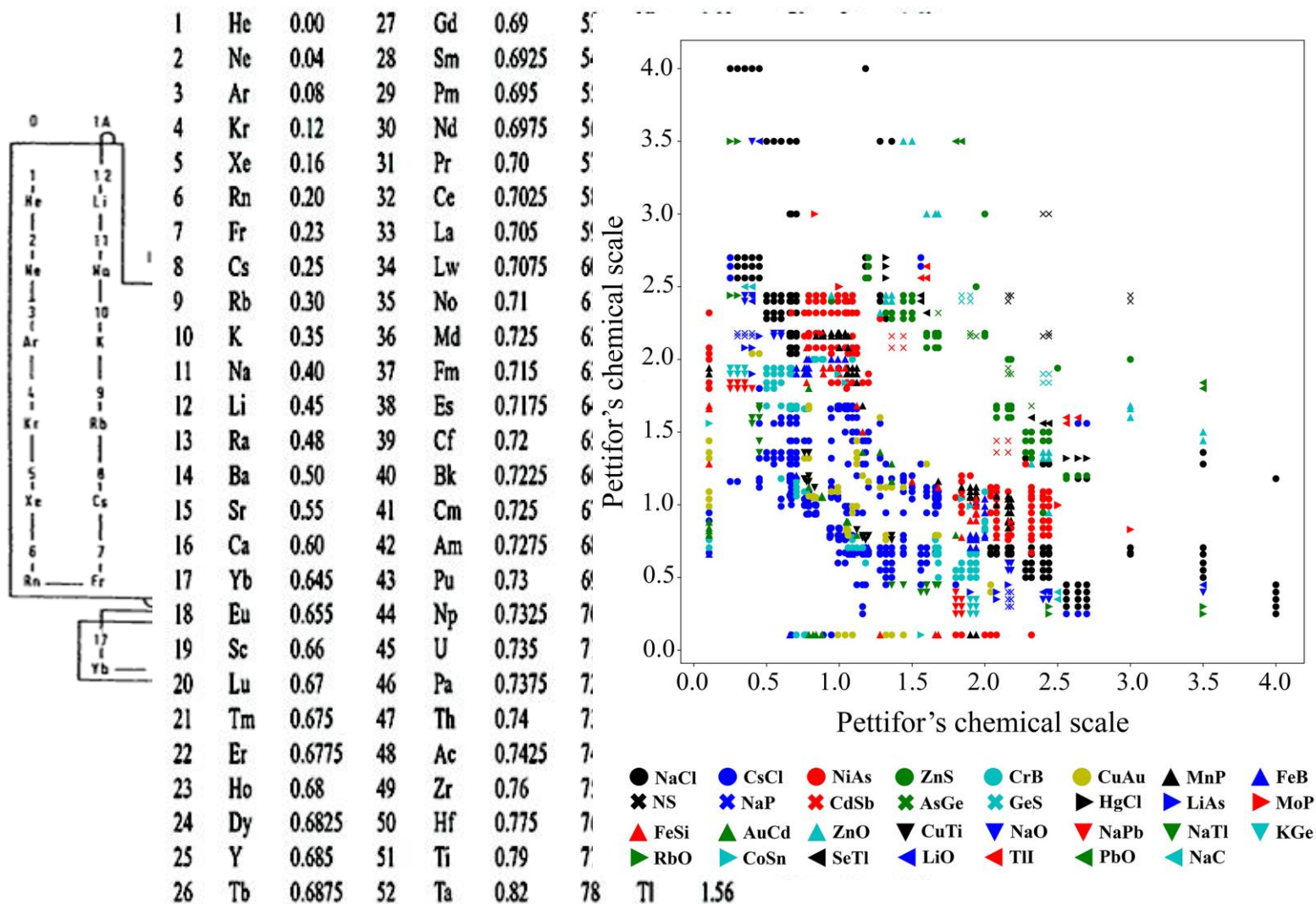
Figure 1: Cooperative coevolutionary architecture from the perspective of species number one.

# Child making in evolutionary algorithm is so obvious, but what about coevolutionary algorithm?



What is the child of  
C-Fe & P-W  
?

# Chemical scale suggested by Pettifor (1984)



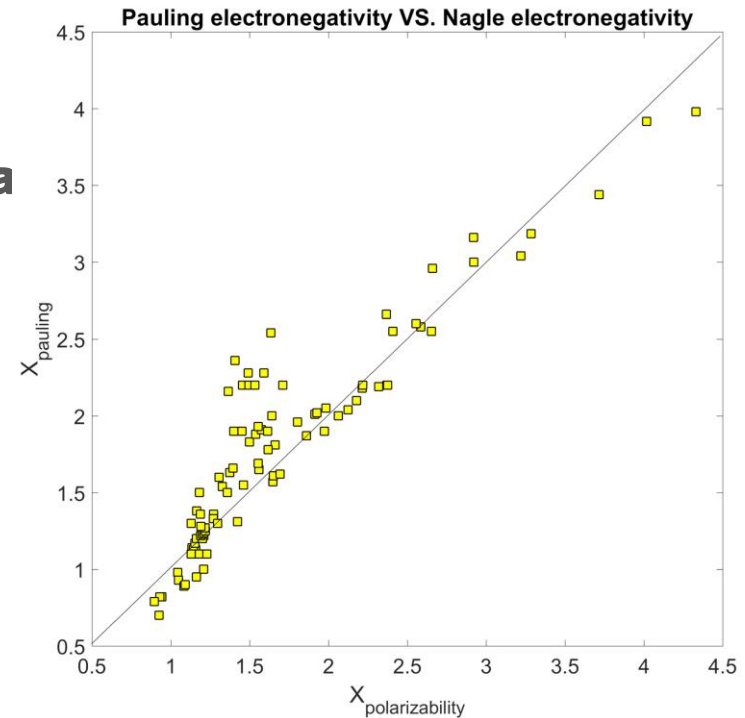


- 1929 - Goldschmidt's law of crystal chemistry: the crystal structure is determined by stoichiometry, **atomic size**, and **polarizability** of atoms/ions [1].
- 1932 - Linus Pauling introduced the concept of **electronegativity** [2].
- 1955 - Ringwood modification: **electronegativity** is as another important parameter for determining the crystal structure [3].
- 1990 - Nagle: **electronegativity** and **polarizability** are strongly correlated [4].

## Organizing the chemical

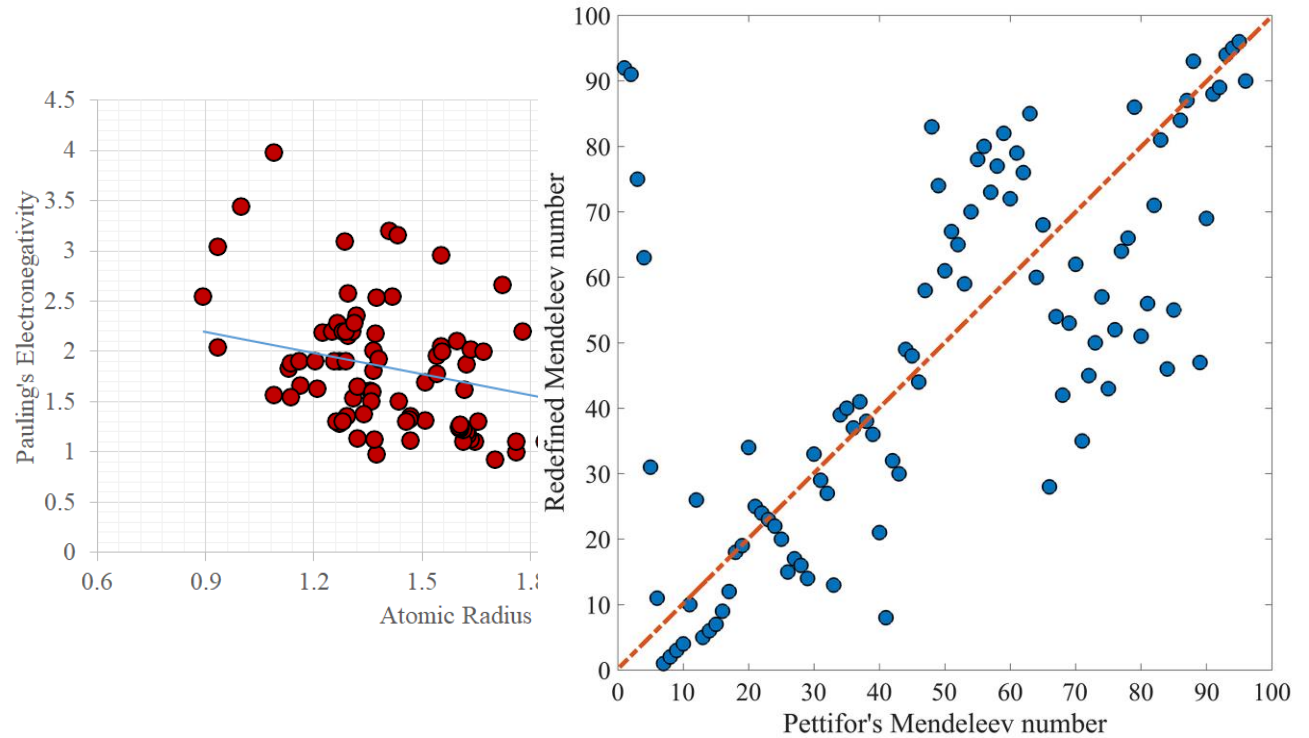
$$\chi_{\alpha} = 1.66(n/\alpha)^{1/3} + 0.37$$

$\chi_{\alpha}$  = electronegativity,  $\alpha$  = polarizability &  $n$  = valence.

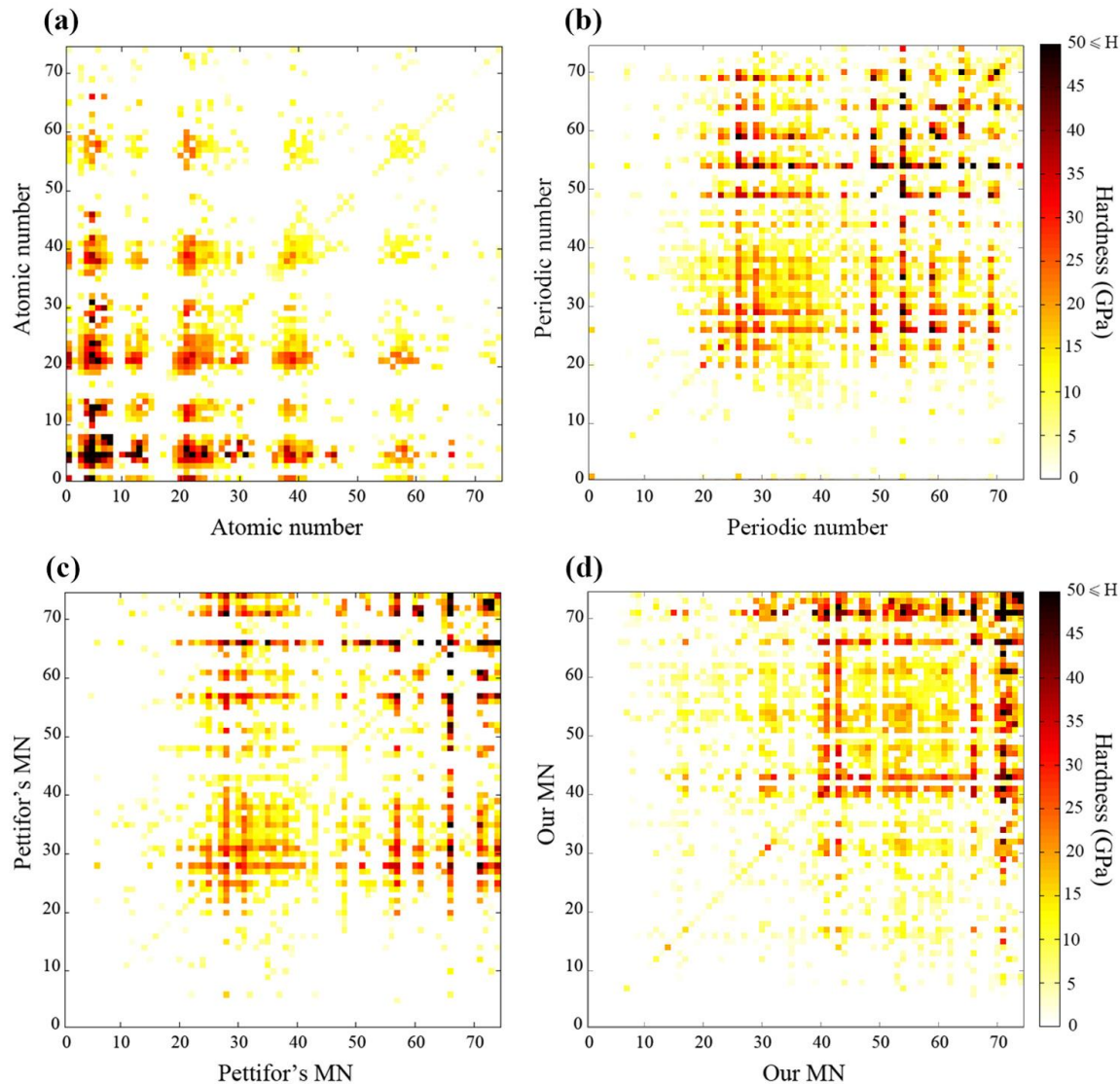




# Our redefined MN, and it's comparison with the Pettifor's MN.

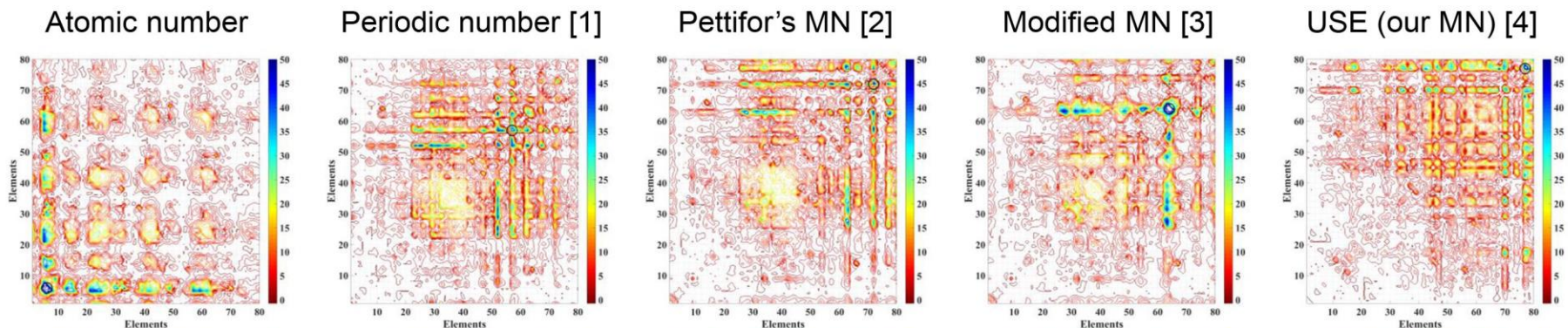


# Pettifor maps of Lyakhov-Oganov model of hardness for different MNs

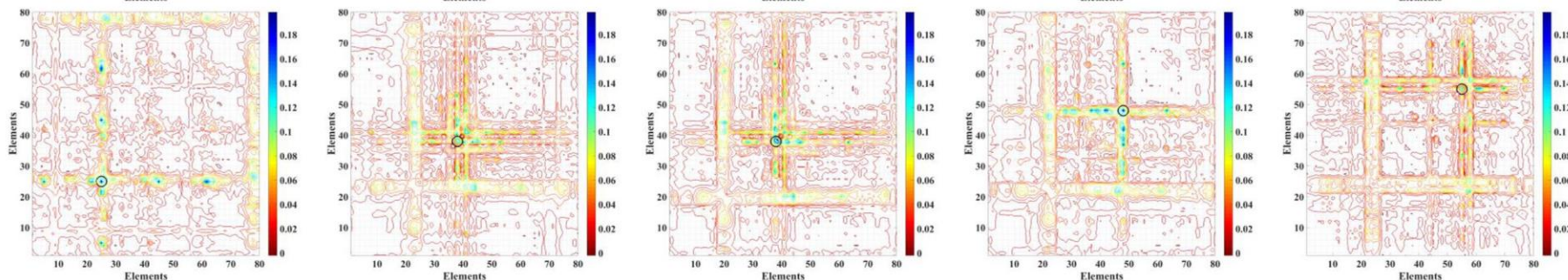




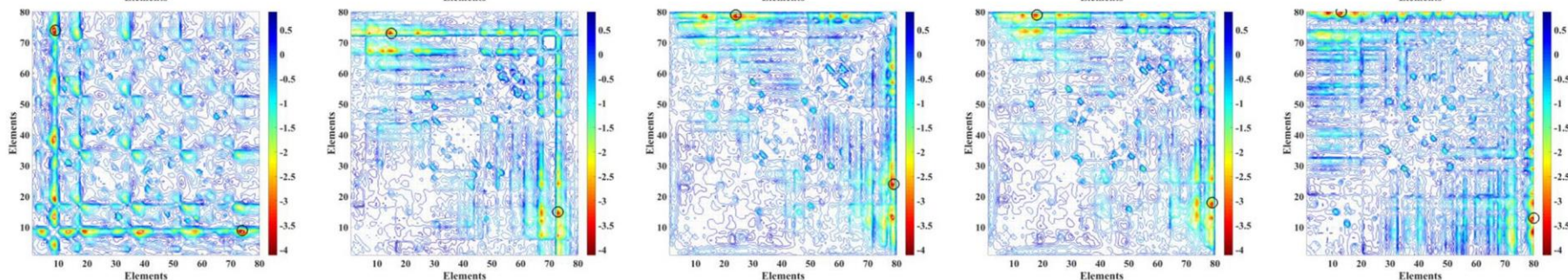
Hardness



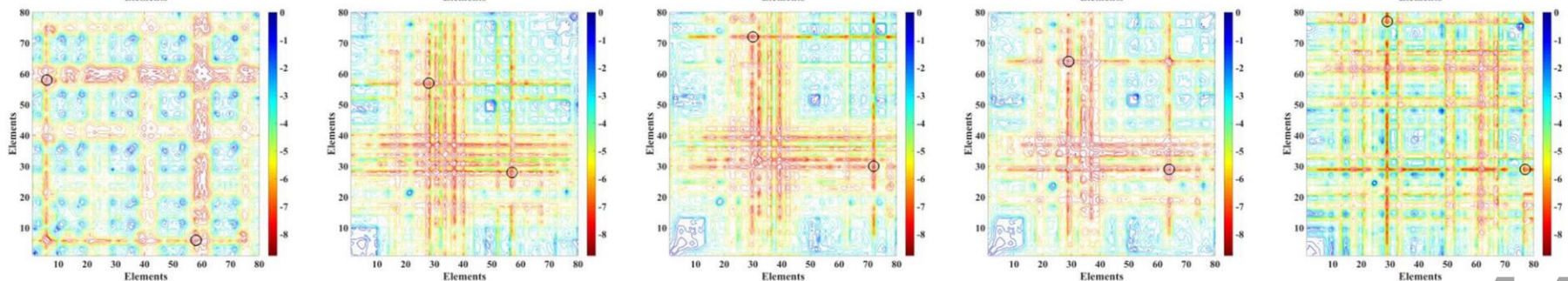
Magnetization



Enthalpy of formation



Atomization Energy



[1] *Chem. Met. Alloy.* **1**, 1-23 (2008).

[2] *J. Phys. C Solid State Phys.* **19**, 285 – 313 (1986).

[3] *New J. Phys.* **18**, 093011 (2016).

[4] *J. Phys. Chem. C*, **124**, 43, 23867–23878 (2020).



## Clustering using density peaks

Science, **344**, 1492 – 1496 (2014).

$$\text{Local density}(\rho_i) \quad \rho_i = \sum_j \chi(d_{ij} - d_c)$$

$d_c$  = cutoff distance

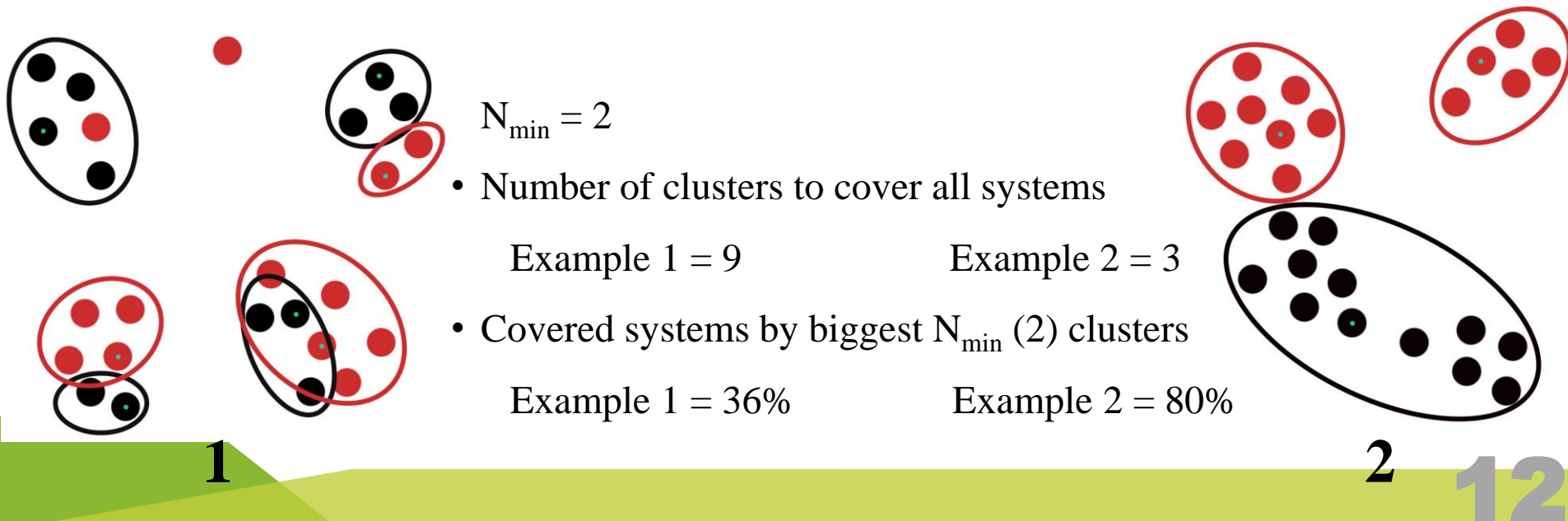
$\chi(x) = 0$  if  $x < 0$

$\chi(x) = 1$  if  $x > 0$

Cluster centers are points with highest local density.

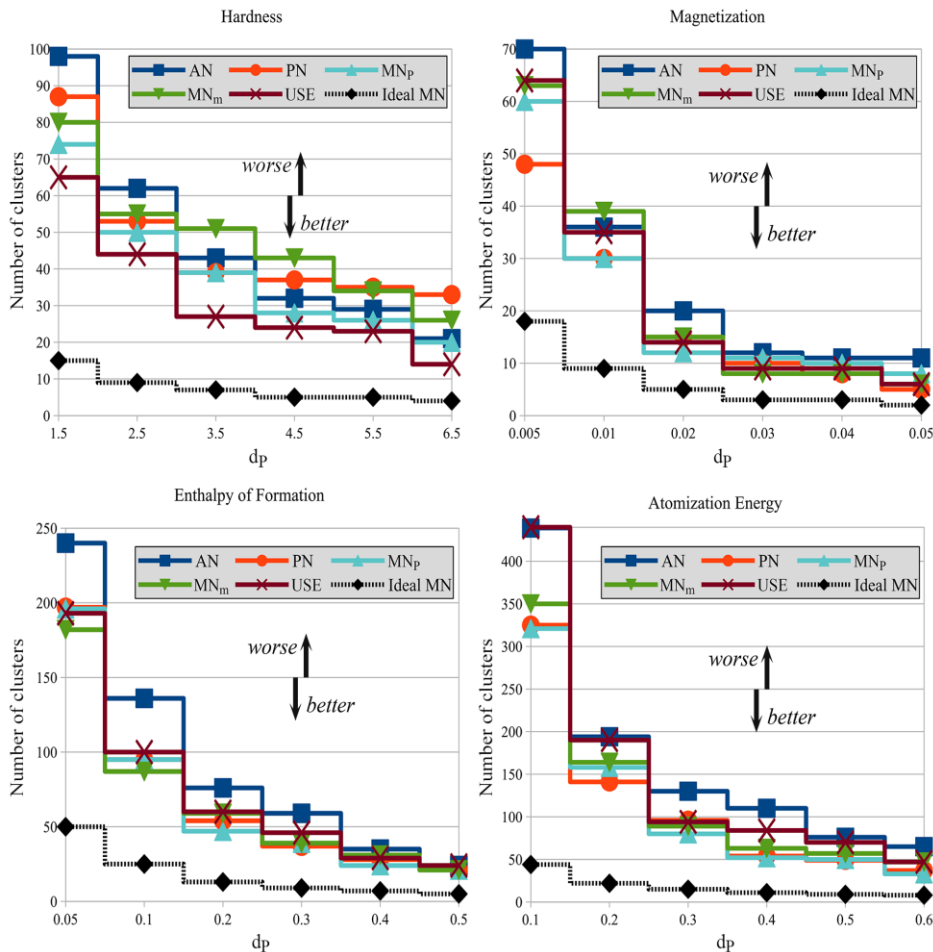
$d_p$  = property difference cutoff between a cluster member and the cluster center.

Ideal MN = an imaginary MN, that covers all systems in minimum number of clusters ( $N_{\min}$ ).

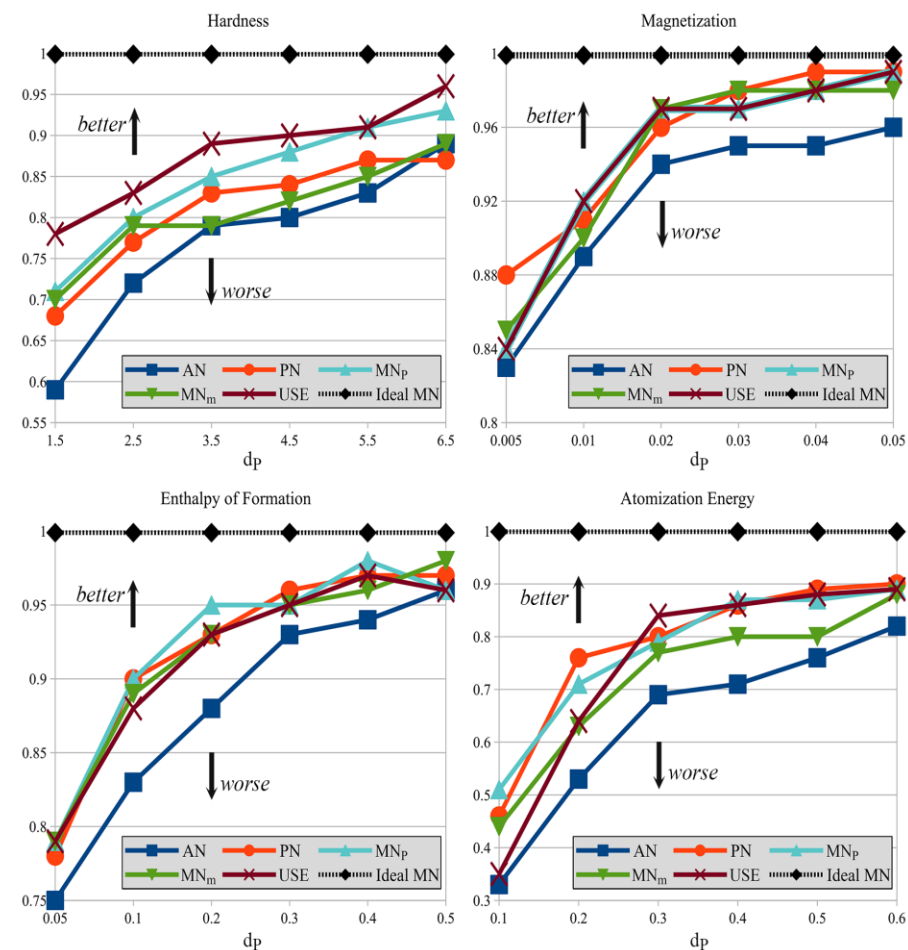


# Evaluation of the MNs using clustering method

Number of clusters

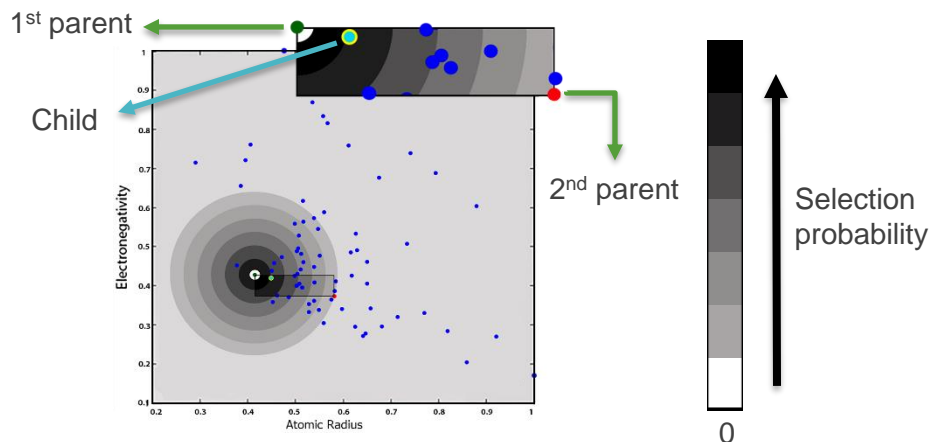


Fraction of systems that are covered by  $N_{min}$  number of clusters

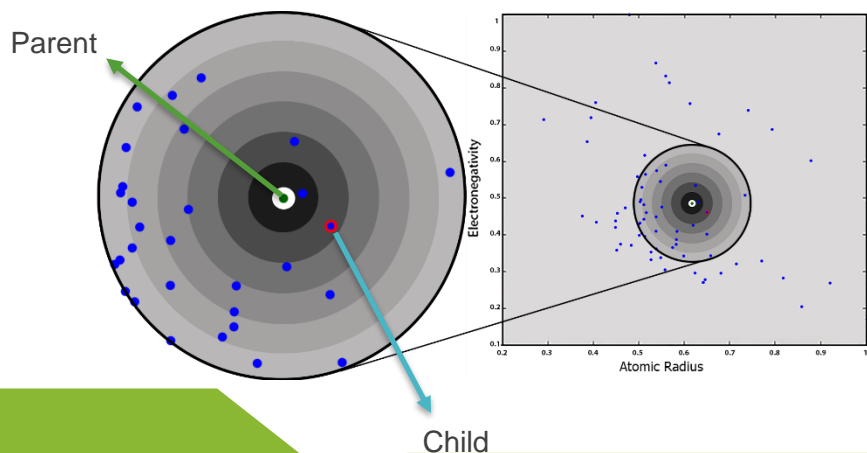


# MendS algorithm and its important Variation operators

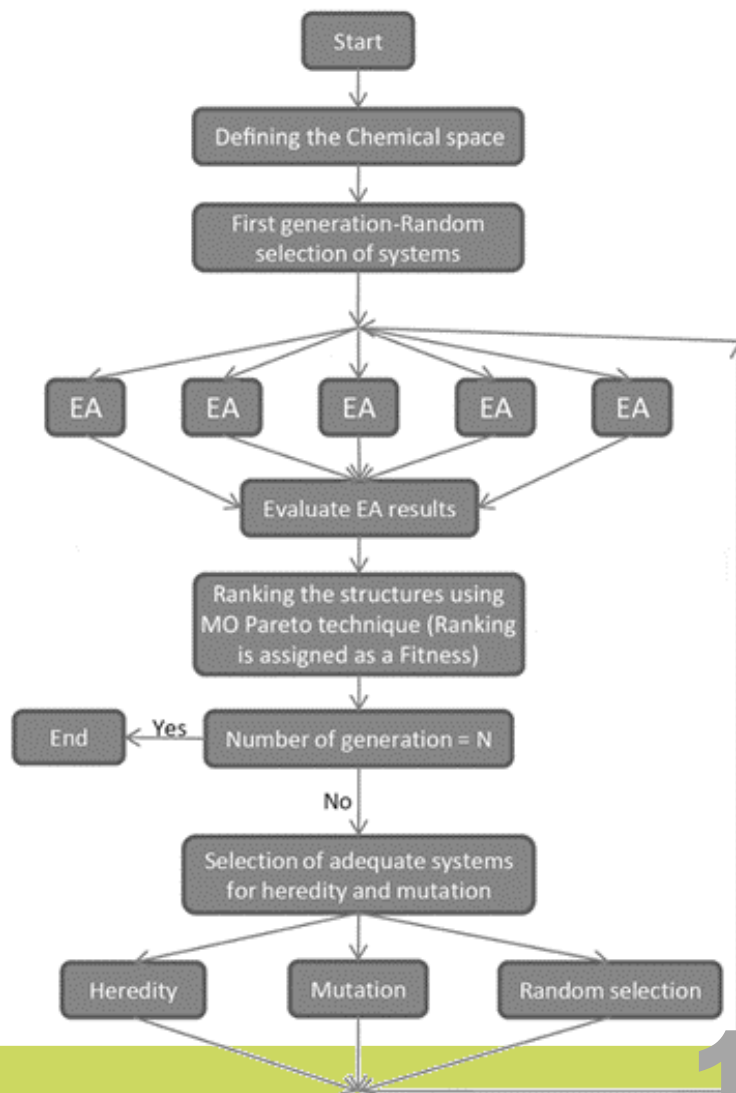
Finding the child of two parent red and green using **chemical heredity operator**.



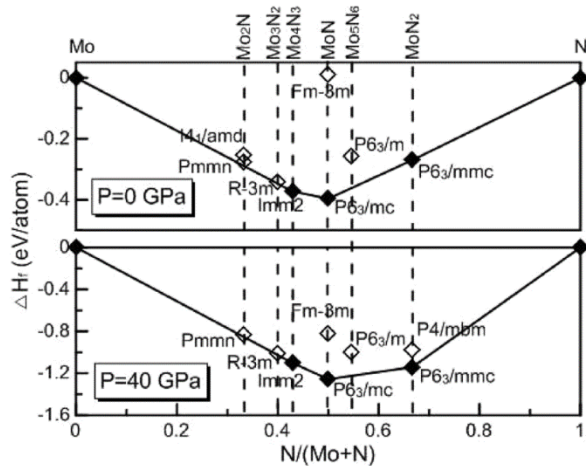
Using **chemical mutation operator** we mutate an atom to find



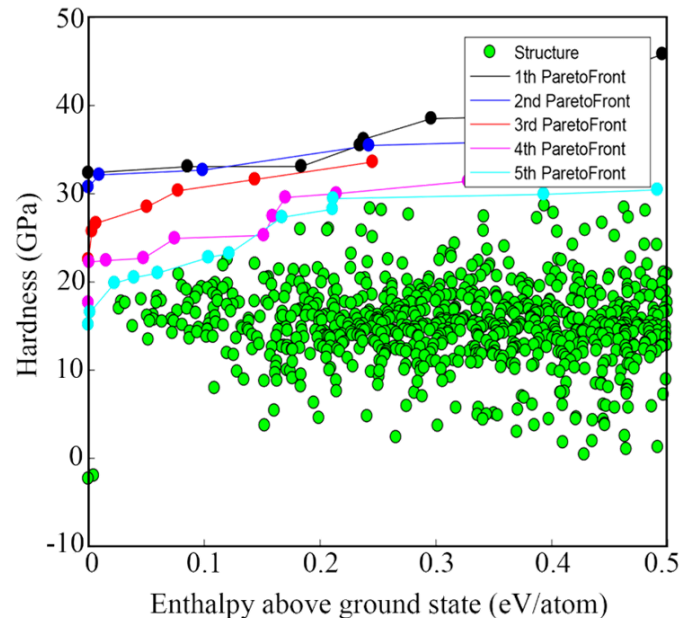
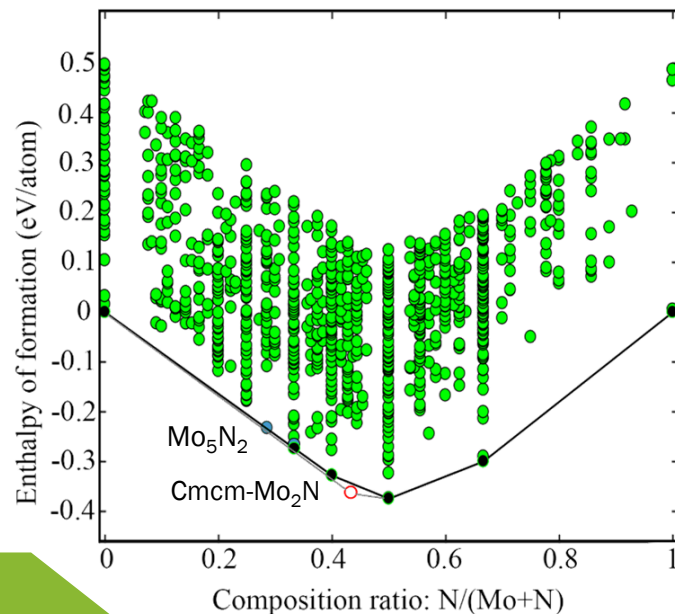
**Reactive heredity operator:**  
A-B & C-D → A-C , A-D, B-C or B-D



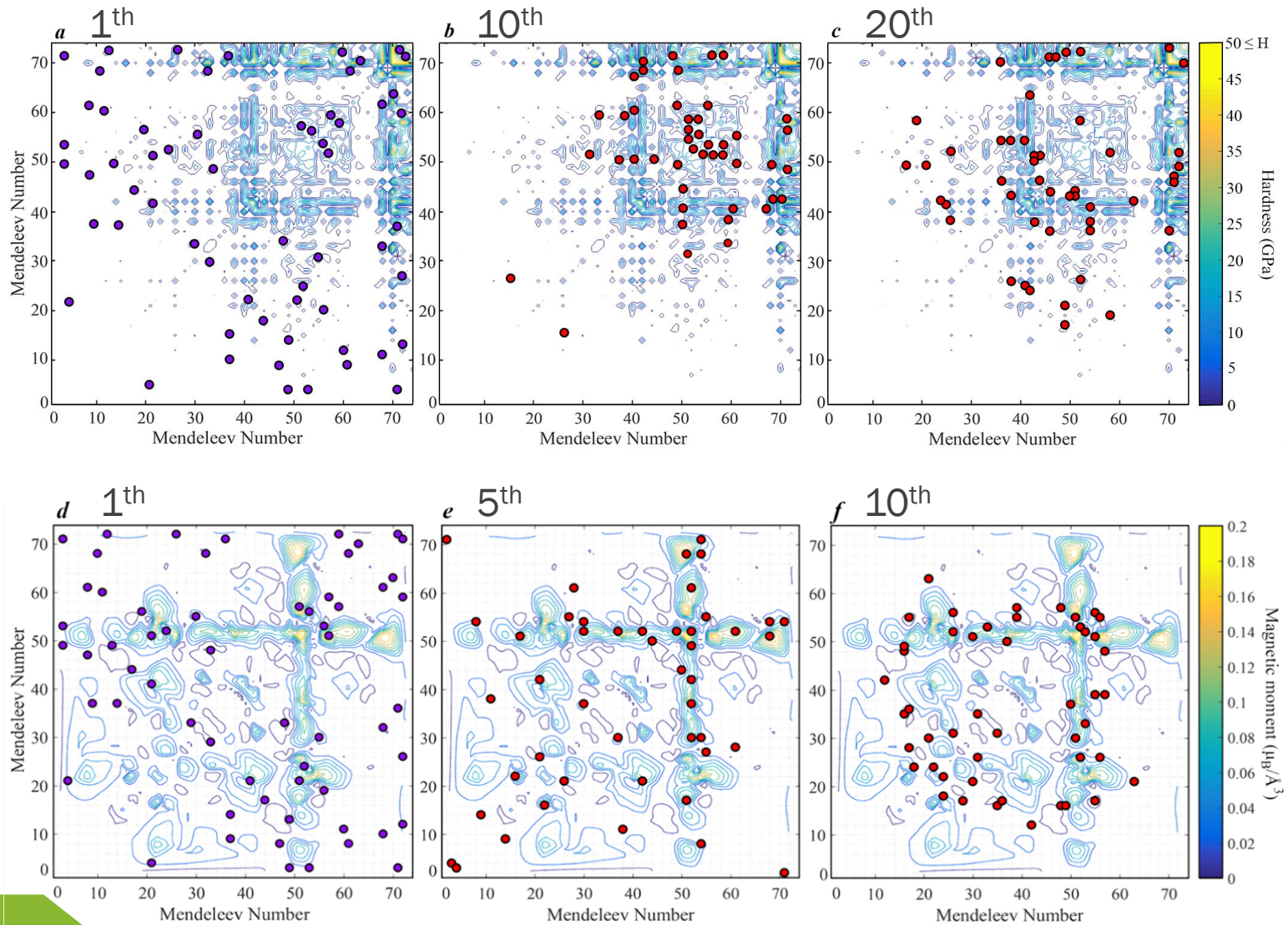
# Implementation of multi-objective Pareto method and its test on $\text{Mo}_x\text{N}_y$



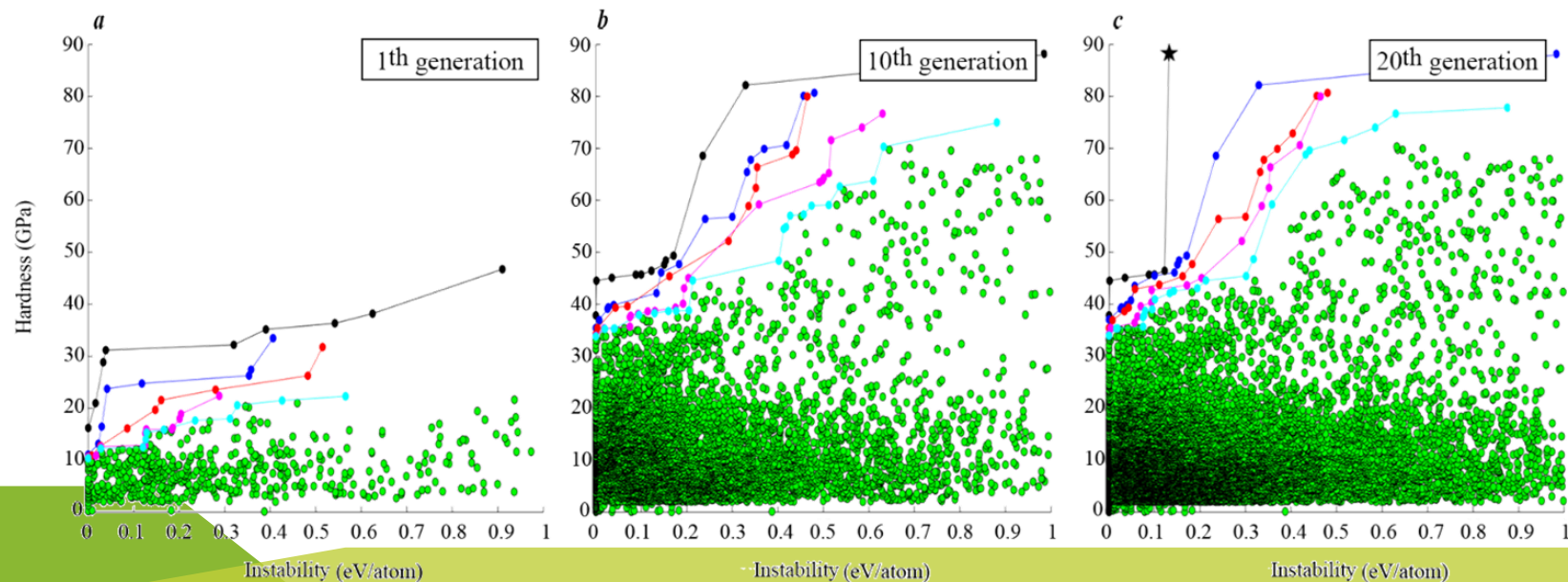
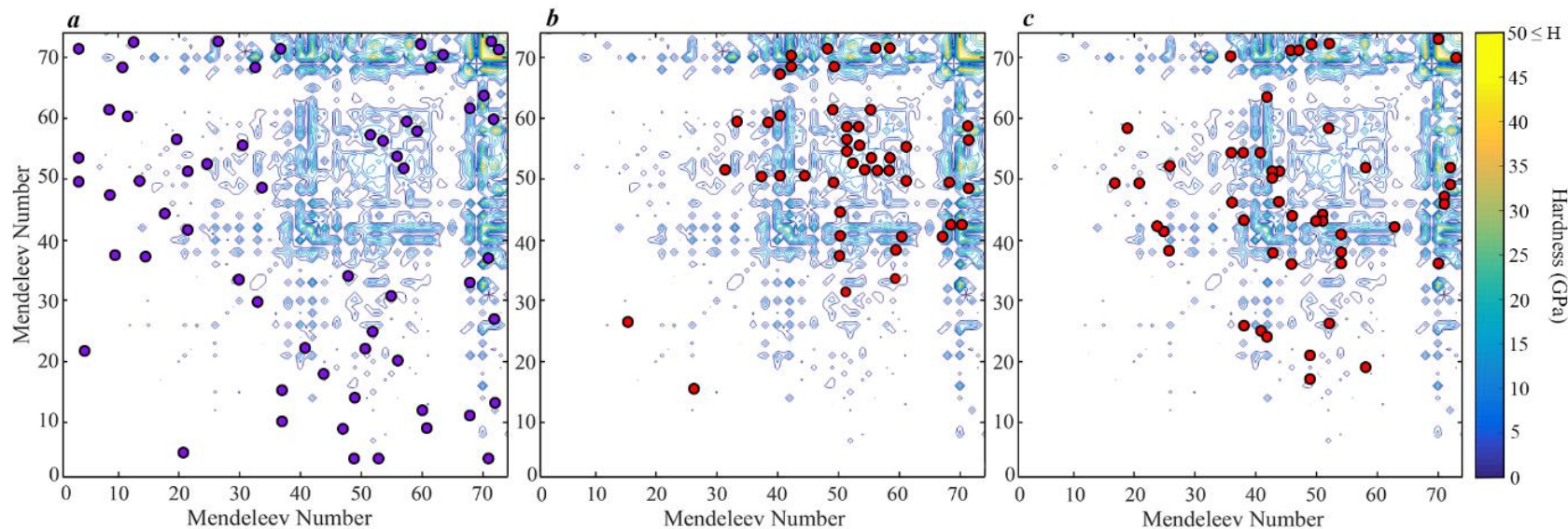
*J. Phys. Chem. C*, 120 (20), pp 11060–11067 (2016).








# Efficiency of the algorithm in the system selection – improving the Pareto front



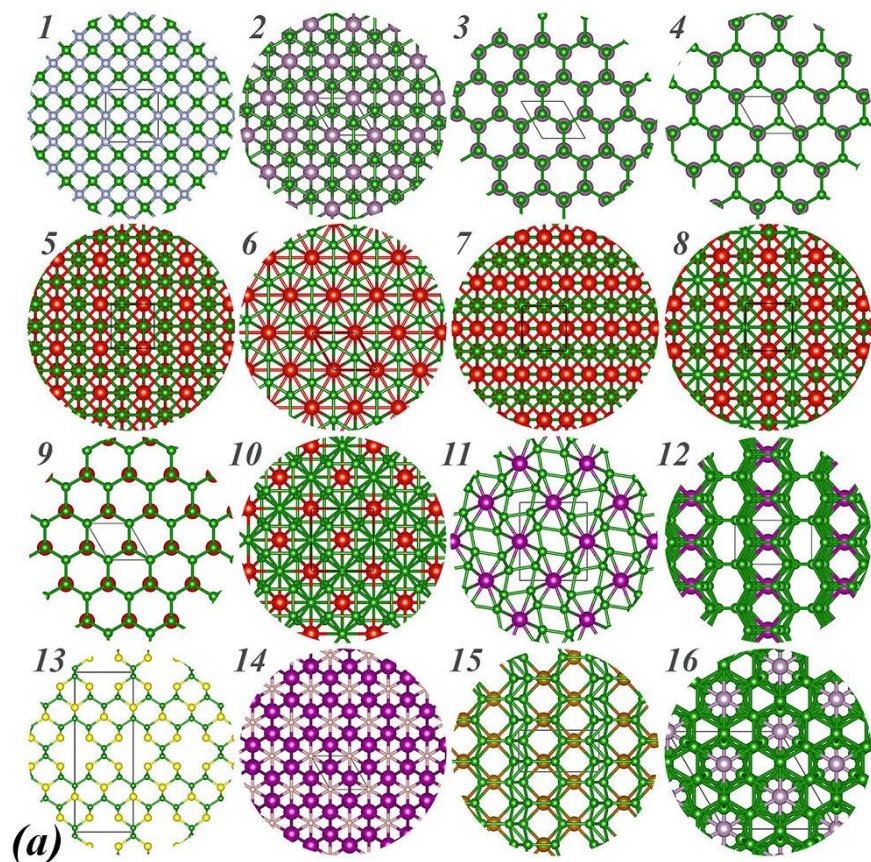
	Compounds	$H_v$ (GPa)	$K_{1C}$ (MPa.m <sup>1/2</sup> )	Instability (eV/atom)	Space group		Compounds	$H_v$ (GPa)	$K_{1C}$ (MPa.m <sup>1/2</sup> )	Instability (eV/atom)	Space group
<b>Carbon</b>	<i>C</i>	<b>92.7, (93.6), [96]</b>	6.33	0.13	<i>Fd3m</i>	<b>Boron</b>	<i>B</i>	38.9, (39), [27-34]	2.87	0	<i>R3m</i>
	<i>C</i>	<b>93.6</b>	6.36	0.139	<i>P6<sub>3</sub>/mmc</i>		<i>B</i>	<b>44.8</b>	3.29	0.136	<i>Cmc2<sub>1</sub></i>
<b>B-S</b>	<i>B<sub>4</sub>S<sub>3</sub></i>	30.5	1.83	0.102	<i>Cmcm</i>	<b>B-N</b>	<i>BN</i>	<b>63.4, (62.8)<sup>25</sup>, [46-80]<sup>20</sup></b>	5.1	0.075	<i>F43m</i>
<b>Mo-B</b>	<i>MoB<sub>2</sub></i>	28.5,(33.1) <sup>44</sup> ,[24.2] <sup>53</sup>	3.76	0	<i>R3m</i>	<b>Tc-B</b>	<i>TcB</i>	31,(30.3) <sup>54</sup>	3.83	0.013	<i>P3m1</i>
	<i>MoB<sub>3</sub></i>	35.3	3.74	0.035	<i>P3m1</i>		<i>TcB<sub>3</sub></i>	27.2,(29) <sup>55</sup>	3.6	0	<i>P6m2</i>
	<i>MoB<sub>3</sub></i>	32.2	3.63	0.077	<i>A2/m</i>		<i>TcB<sub>3</sub></i>	33.1	3.79	0.003	<i>P3m1</i>
		35.3,(37.3) <sup>44</sup>	3.63	0.017	<i>P6<sub>3</sub>/mmc</i>		<i>TcB<sub>4</sub></i>	31.8	3.56	0.069	<i>P2<sub>1</sub>/m</i>
		33.1,(31.8) <sup>43</sup>	3.57	0.011	<i>R3m</i>		<i>TcB<sub>4</sub></i>	30.2	3.54	0.069	<i>R3m</i>
	<i>MoB<sub>4</sub></i>	35.4	3.57	0.099	<i>Pmmn</i>			30,(32) <sup>55</sup>	3.57	0.027	<i>P6<sub>3</sub>/mmc</i>
	<i>MoB<sub>5</sub></i>	35.7	3.62	0.054	<i>P6m2</i>		<i>TcB<sub>7</sub></i>	35.9	3.35	0.084	<i>R3m</i>
	<i>MoB<sub>8</sub></i>	36.6	3.24	0.118	<i>R3m</i>		<i>TcB<sub>8</sub></i>	33.9	3.3	0.113	<i>R3m</i>
	<i>Mo<sub>2</sub>B<sub>3</sub></i>	32.2	3.95	0.029	<i>Immm</i>		<i>Tc<sub>3</sub>B<sub>5</sub></i>	30.6	3.87	0	<i>P6m2</i>
	<i>Mo<sub>2</sub>B<sub>3</sub></i>	30.4	3.87	0.043	<i>Cmcm</i>						
<b>Si-C</b>	<i>SiC</i>	33.3,(33.1) <sup>31</sup> ,[28] <sup>31</sup>	2.94	0	<i>F43m</i>	<b>B-P</b>	<i>BP</i>	37.2,(29.3) <sup>31</sup> ,[33] <sup>31</sup>	2.46	0	<i>F43m</i>
	<i>SiC</i>	33.1	2.94	0.001	<i>R3m</i>		<i>B<sub>6</sub>P</i>	<b>41.1</b>	2.87	0	<i>R3m</i>
<b>V-B</b>	<i>VB</i>	39.1,(38.3) <sup>49</sup>	3.66	0	<i>Cmcm</i>	<b>Mn-H</b>	<i>MnH</i>	29.5	3.2	0	<i>P6<sub>3</sub>/mmc</i>
	<i>VB<sub>2</sub></i>	37.3,(39.5) <sup>49</sup> ,[27.2] <sup>50</sup>	3.75	0	<i>P6<sub>3</sub>/mmm</i>		<i>MnH</i>	27.9	3.14	0.013	<i>R3m</i>
	<i>VB<sub>5</sub></i>	<b>40</b>	3.36	0.158	<i>P6m2</i>		<i>MnH</i>	26.3	3.07	0.044	<i>Fm3m</i>
	<i>VB<sub>7</sub></i>	39.7	3.19	0.143	<i>P3m1</i>		<i>Mn<sub>3</sub>H<sub>2</sub></i>	26.8	3.22	0.017	<i>R32</i>
	<i>VB<sub>12</sub></i>	<b>44.5</b>	3.34	0.125	<i>I4<sub>1</sub>/mmm</i>		<i>Mn<sub>3</sub>H<sub>2</sub></i>	27	3.26	0.019	<i>P6<sub>3</sub>/mcm</i>
	<i>V<sub>3</sub>B<sub>4</sub></i>	37.8	3.74	0	<i>P4m2</i>		<i>Mn<sub>4</sub>H<sub>3</sub></i>	27.6	3.23	0.002	<i>P2<sub>1</sub>/m</i>
	<i>V<sub>3</sub>B<sub>4</sub></i>	35.9,(38.2) <sup>49</sup>	3.7	0.006	<i>Immm</i>		<i>Mn<sub>6</sub>H<sub>5</sub></i>	27.3	3.17	0.011	<i>A2/m</i>
<b>Mn-B</b>	<i>MnB<sub>3</sub></i>	32.2	3.5	0.029	<i>P6m2</i>	<b>Fe-B</b>	<i>FeB<sub>3</sub></i>	30.2	3.32	0	<i>P2<sub>1</sub>/m</i>
	<i>MnB<sub>4</sub>†</i>	<b>40.7</b>	3.65	0.009	<i>Pnnm</i>		<i>FeB<sub>4</sub></i>	35.7	3.06	0.021	<i>Immm</i>
	<i>MnB<sub>4</sub></i>	38.2	3.56	0.1	<i>R3m</i>		<i>FeB<sub>4</sub>†</i>	32	3.31	0.039	<i>R3m</i>
		38.1,(40.5) <sup>51</sup> ,[37.4] <sup>52</sup>	3.76	0	<i>P2<sub>1</sub>/c</i>		<i>FeB<sub>4</sub></i>	<b>42.7</b>	3.31	0.063	<i>A2/m</i>
	<i>MnB<sub>5</sub></i>	32.7	3.38	0.097	<i>P6m2</i>			28.6,(24.4) <sup>61</sup> ,[62] <sup>59</sup>	3.32	0.002	<i>Pnnm</i>
	<i>MnB<sub>13</sub></i>	<b>40.4</b>	2.9	0.181	<i>Pm</i>		<i>Fe<sub>2</sub>B<sub>11</sub></i>	33.8	3.37	0.081	<i>Pm</i>



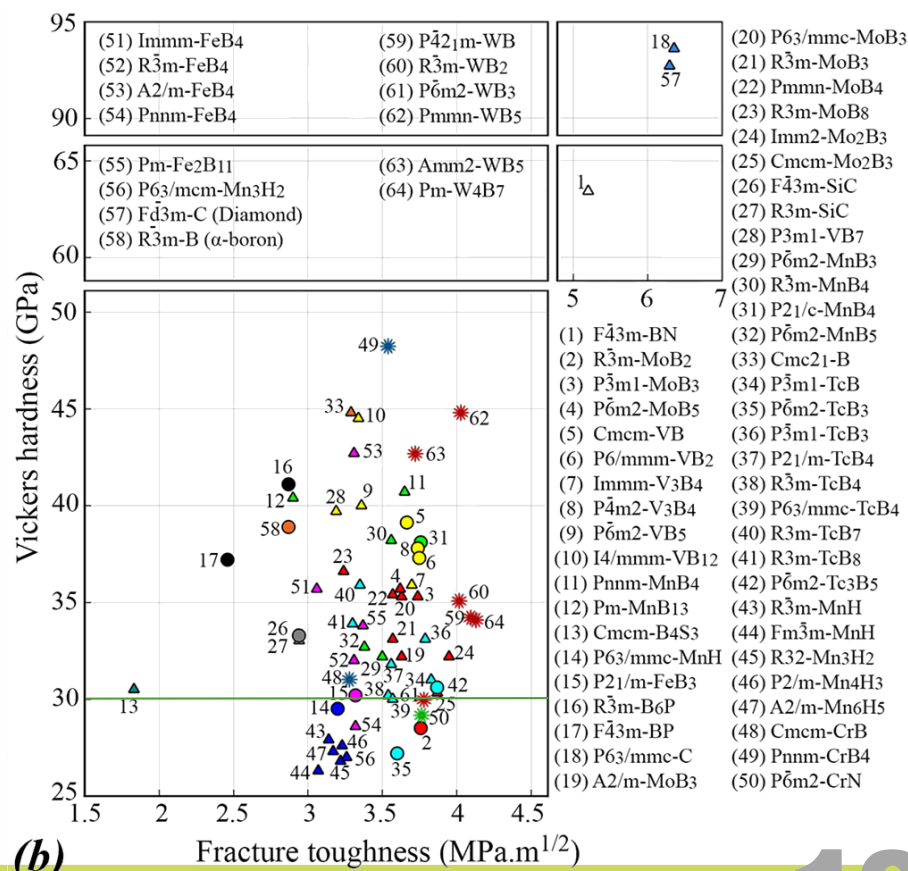
# Ashby plot of hardness vs. toughness for predicted phases

 *J. Phys. Chem. Lett.*, 8 (4), pp 755–764 (2017). Cr-N, Cr-B

 *J. Phys. Chem. Lett.*, 9 (12), pp 3470–3477 (2018). W-B



Tungsten carbide WC  
Hardness 30 GPa



## Conclusion

- A well-defined chemical space can be obtained using the most significant properties of elements (electronegativity and atomic size and binary systems with similar properties are nearby in this space).
- Multi-objective Pareto technique makes it possible to search for materials optimal in more than one property, and works efficiently at least for two properties.
- Combination of coevolutionary algorithm, MO Pareto technique and well-structured chemical space - MendS – works efficiently in searching for materials optimal in multiple target properties.
- Our Mendevian search suggests that diamond is the hardest material (among binaries) in nature – harder cannot be found.
- MendS works at arbitrary pressure and for binary, ternary systems.

Thank you for your attention!

Any questions?

